CONCEPTUAL EXPLICATIONS OF THE DESIGN PROCESS

This content is intended to fill out and add some further explanation to the design process adumbrated in *Mind-Blowing Kusudama Origami*. First, as a caveat, I would note (as others have before) that a written language is not a very compact means for expressing origami concepts. Therefore, describing origami geometry in language tends to be wordy and difficult to conceptualize, so any attempts at clarifying the design process described in *Mind-Blowing Kusudama Origami* will have natural limitations. I have attempted to keep mathematical formulation to a minimum here, focusing rather on the conceptual highlights. Second, this theoretical work on the nature of 2-fold rotationally symmetric origami unit design from square sheets of paper is still in its nascent stage, and is thus far from exhaustive. As with all of the work I do in modular origami, my intention here is to cultivate the creative design process in myself and others, and to expand upon these themes touched upon in *Mind-Blowing Kusudama Origami*. Therefore, feel free to expand or revise any of the concepts here for yourself, to find a method that works for you.

Q: What are some of the preliminary constraints which are applied to this modular origami design process?

A: This modular origami unit design process 1) begins with a square piece of paper 2) is constrained to units whose creases possess 180-degree rotational symmetry around a central point, which is representative of the origin as the intersection of the baseline and the ridgeline, the central point of the paper, and the midpoint of the edge of a polyhedron which the units are modeling 3) contains no cuts or adhesives 4) follows the rules of flat foldability except where such flat foldability conflicts with a)3D embellishments or b)regions of the paper that will remain 3D during the assembly process and 5) is oriented towards more classical kusudama design paradigms, emphasizing the scope of the standard "tab and pocket" style modules. Therefore, this design paradigm is not optimized for the design of novel methods of assembly, non-polyhedral shapes or exotic locking mechanisms.

Q: What is a Polygon Base?

A: Kusudamas traditionally model polyhedral structures. Every self-enclosed polyhedral structure (that is, a structure with convex polyhedral symmetry containing equal length edges and no degenerate polygonal faces) modeled in an origami kusudama has specified regions of paper from the starting squares of paper of each folded unit which show on the interior of the model as the basic "shape" of the unit, that is, the representative region of a singular unit which projects a tiling onto the surface of a maximal volume inscribed sphere. These represent a) the 2-fold axial projections of the zonohedral spherical tiles of the polyhedral shape represented by the units (for example, the projection of a unit's visible perimeter onto one of the rhombuses of a rhombic triacontahedron in the case of an icosahedral assembly modeling the edges of an icosahedron), and also b) the uninterrupted central 2-fold polygonal region of the paper which is symmetrically divided by the Baseline and Ridgeline of the unit. I refer to the latter as an Identity Polygon Base, because it fundamentally constrains the central design of the unit pattern, and the former as an Augmented Polygon Base, because it includes the entire perimeter of the unit as viewed from inside of the model, and thus illustrates each unit's contribution to the completed structure. The Identity Base is quite simply the polygon which lies at the center of the unit in the unfolded crease pattern. Kusudama units are rarely flat (as in, possessing entirely unfolded Baselines or Ridgelines), so genuine orthogonal projections would usually be distorted, and unnecessarily difficult to calculate mathematically in practice. Therefore, the notion of projection here is strictly theoretical—in practice, the Augmented Polygon Base is simply the perimeter of the unit as you would visualize it if you were looking outward from inside of the unit. In all cases, the crease patterns of both types of Polygon Bases will maintain centrally symmetric point reflections about the origin of the Standard Basis, which includes both the Baseline and Ridgeline. By the rules of flat foldability, any interior polygon base angles >180 degrees will entail that those angles are part of the Augmented Polygon Base. In other words, only the Augmented Polygon Base can possess concave angles. In many units, the Identity and Augmented Polygon Bases are identical. An Identity Base can have a partially folded line traversing it, but cannot have overlapping layers. Thus, some Identity Bases are "degenerate digons," but no Augmented Bases are "degenerate digons," because some polygon bases have layers converging on the baseline on the underside of the unit (from my book, Tuscia would be an example of this). This is not to say that the Identity Base as visualized on the unfolded crease pattern is simply a line (although this is indeed sometimes the case), but rather that in a completed unit, there exists no uninterrupted region of paper which is both visible and constitutes the exterior

contribution of each unit to the kusudama. Polygon Bases constrain the range of potential designs and assemblies, correlate to the paper-area efficiency of the unit, and constrain the range of reference creases for a kusudama unit representing the edges of a polyhedron.

Q: If a tab of an adjacent unit lies on top of a polygon, how does this affect the Augmented Polygon Base?

A: Tabs which appear as part of the Augmented Polygon Base can sometimes fill in negative space of an adjacent unit's Augmented Polygon Base. From my book, Carpathia would be an example of such a unit. In such cases, the Augmented Polygonal Base can visually extend into the 2-fold axial space of what would be a neighboring zonohedral spherical tiling region from the inscribed sphere. In the case of Carpathia, the model appears to have an irregular octagonal Augmented Polygon Base, but the notches on the pocket sides of the unit result in regions of the tabs of an adjacent unit which cover the notches. Thus, the Augmented Polygon Base in this model is partially constituted by what would traditionally be considered part of the tabs.

Q: How does the Identity Polygon Base relate to the Augmented Polygon Base?

A: The entire visual area of the unit (as in, what you see of each unit on a completed assembly) is included in the 2-fold projection, but the Augmented Polygon Base is the region of the unit which is visible from the interior of the construction, the paper that is both integral to the structure of the model (because it defines the perimeter of the units) and that which is not overlapping any other units. Every polyhedron used for kusudama construction can be divided into rhombic zonohedral regions aligning with the polyhedron's edges, and the flattened projection of the interior of the edge unit on to its corresponding rhombic region is the projection of the Polygon Base. If this is composed of a single uninterrupted region of paper from the original starting square, the Base will be an Identity Base, whereas, if this projected area includes other layers, it will be an Augmented Base. The Identity Base in a unit design is thus a subset of the Augmented Base, which is a subset of the unit perimeter.

Q: How does the Augmented Polygon Base relate to the perimeter of the folded unit?

A: Expanding from the Augmented Polygon Base, the perimeter of the folded unit includes the tabs and any embellishments which project outwards to form the perimeter of the folded unit. As noted on page 13 of *Mind-Blowing Kusudama Origami*, extrinsic rotations of the baseline and ridgeline around a unit in fixed orientation can utilize the full perimeter of the folded unit. This means that as long as two of the Polygon Base's vertices lie on the Baseline, the Baseline or Ridgeline can be arbitrarily rotated. Also note that unlike the rotations on the original square of paper which had four-fold rotational symmetry, extrinsic rotations on the perimeter of a folded unit with existing 2fold symmetry will have 180 degrees of rotational freedom in units which are not reflectionally symmetrical across their existing ridgelines.

Q: What are the Baselines and Ridgelines? Are they arbitrarily chosen?

A: A square piece of paper is here defined as in a standard orientation when the midpoints of the sides of that square lie on the x and y axes at any arbitrarily defined scalar value. Put otherwise, a square of paper is in standard orientation when its edges are horizonal and vertical to the viewer. The Baseline and Ridgeline initially lie on the Standard Basis (in R2) which divides the paper into four orthogonal quadrants. Thus, the x and y axis are often referred to in mathematics as the Standard Basis, and the x axis here is referred to as the Baseline, while the y axis is referred to as the Ridgeline. Distinguishing between intrinsic and extrinsic rotations, it is possible to keep the paper unmoved and rotate the coordinate frame of reference (the Baseline and Ridgeline) up to 90 degrees. A rotation of 90 degrees or more will result in the Baseline and Ridgeline reiterating their relationship on the next corner around on the square in which direction the frame of reference was rotated. By definition, the Baseline is arbitrarily chosen as the standard orientation of the unit, just as the notion of the x-axis as horizontal and the y-axis as vertical is arbitrary. By the conventions used in my book, the Baseline is the standard folding sequence's primarily utilized line of symmetry, and the Baseline will always pass through two vertices of the Polygon Base. For example, in the variation of Galicia, the unit's polygon base appears as a rectangle, but is technically a hexagon, since the long edges of the rectangle are interrupted by the Baseline at the center of the unit.

Q: Both the Baseline and Ridgeline are frequently referenced, as well as the Standard Basis: what is the difference between these terms?

A: The term "Standard Basis" and references to the "Baseline and Ridgeline" are interchangeable in this context. Standard Basis is a mathematical term referring to vectors which are identical to the x and y axes, the scaled linear combination of which are capable of generating all real vectors in R2. As noted in *Mind-Blowing Kusudama Origami*, the Standard Basis divides the starting square of the paper into four equally sized and shaped regions of paper, and within ≤2 of these regions, all unique design decisions will occur in radial, centrally symmetric kusudama unit designs. The two lines which divide the paper into four regions are the Baseline and Ridgeline. These two lines are orthogonal/perpendicular to each other.

Q: What is meant by the concept that all creative design decisions take place within ≤2 quadrants of the Standard Basis?

A: In a unit which is 2-fold rotationally symmetric, all unique creases will exist within ≤ 2 quadrants of the Standard Basis. In most 2-fold rotationally symmetric edge units, two adjacent quadrants (it does not matter which two are chosen, so long as they are adjacent) will contain all of the paper within which a design can arbitrarily add new folds. This pattern of folds within any two adjacent quadrants is then copied and rotated 180 degrees around the central point of the paper (the origin of the Standard Basis), which fills in the creases for the other two quadrants. For example, when drawing a crease pattern for the unit, the design may simply draw all of the creases in any two adjacent quadrants (which together constitute 50% of the paper) and "copy and paste" this pattern (in 180-degree rotational symmetry) to the other half of the paper. However, in the case of units which are both reflectionally and rotationally symmetric, a single quadrant of the paper contains all the unique creases of the design. The Caria kusudama from Mind-Blowing Kusudama Origami would be an example of such a model. In the case of such models the properties of both the "tab" and "pocket" components of the locking mechanisms exist in all four quadrants, and the unit's Polygon Base has no reference to "handedness." The designer should note that the assembly of units which are both reflectionally and rotationally symmetric in their crease patterns will often (but not always!) loose this reflectional symmetry in the assembly process. In other words, the reflectionally symmetric unit may establish a handedness in the assembly process, due to the unique functions of identical regions of paper in the assembly process. Again, Caria is an example

from *Mind-Blowing Kusudama Origami* of a model where the unit's reflectional symmetry is technically lost during assembly due to the relationship between adjacent units.

Q: Are the Baseline and Ridgeline constrained by the Polygon Base or do they constrain the Polygon Base?

A: The answer is both: first the lines constrain the base perimeter, then the base perimeter constrains the lines, because wherever the baseline passes through the unit, two vertices of the polygon base will be fixed. The ridgeline will always be fixed as orthogonal to the baseline.

Q: Can the Baseline pass through vertices of an Augmented Polygon Base where there exists only a single separate layer of paper?

A: Yes, it can, and does in models such as Aquitania in *Mind-Blowing Kusudama Origami*.

Q: Are the tabs of a unit design always located at the extremities of the original starting square?

A: No, there are a number of more complex designs where the regions of the paper designated from the original starting square for the tabs and the pockets are not located near the edges of the paper, but such designs do tend to be fairly complex. The disadvantage of such designs is that they take longer to fold, and they tend to accumulate more layers in the tab and pocket regions, which generally reduces the precision of the assembly.

Q: Does the Polygon Base include the negative space formed by windowed region?

A: Any form of the Polygon Base will include only paper which is part of the unit. Augmented Polygon Bases with >4 sides frequently have windowed regions, which include any intentional openings by which the observer may peer into and through the model. These openings project as negative space unto a zonohedral tile corresponding with the edge unit. Given the fact that a 90-degree rotation of the Standard Basis may be unique in a completed unit (in contrast to a rotation of a Standard Basis by 90 degrees in the starting square, which will not be unique), a regular hexagon Identity Polygon Base will have one of the largest potential regions of negative space. Imagine an animation of a standard star kusudama with a 4-sided Identity Polygon Base

where the design transitions to a 6-sided Polygon Base and begins to incorporate small windows at each stellated vertex. As the windows expand, the proportion of negative space expands with respect to the Polygon Base. As the windows pass the $1/4^{th}$ point of the Ridgeline (as in, halfway to the center of the unit), the unit's base may be rotated 90 degrees and scaled to fill the corresponding rhombic spherical tile, thus reducing the resulting negative space.

Q: How do the reference creases relate to the Polygon Base?

A: The vertices are points on the surface of an unfolded square of paper which define the corners of either form of the Polygon Base (they usually define the Identity Base first, as this is much simpler to design). They are chosen within the design space by the intersection of reference creases, and they will define, either directly or indirectly, the shape and the size of either Polygon Base. By definition, ≥ 2 distinct creases will intersect at each vertex of an Identity Polygon Base. In the case of a unique Augmented Polygon Base, two or more vertices will be separated from the Identity Base by creases which do not constitute an edge in the perimeter of either Base, and these creases may be constrained by the reference creases. Reference creases may initially establish a rotational orientation of the Standard Basis, thus constraining the orientation of either Polygon Base. Reference creases may be additive, in the sense that you may use the first creases you make on a square to create a pair of new creases, and use the new creases as references for a third pair of creases, and so on. In practice, keeping the number of reference creases to a minimum reduces the amount of time required to fold a unit, and limits the number of unused creases which show on the completed model.

Q: How are the initial reference creases established?

A: There exist remarkably few initial crease possibilities using square paper and assuming that the creases are referenced and hence duplicatable. All referenced folding sequences beginning from a square will fold the paper in half, either joining opposite edges or opposite vertices. There are two types of rotations. The first is the rotation of the Standard Basis, while the second is the rotation of the creases. This second rotation is the second degree of freedom referenced in the book, which establishes the location of the points which define the size, shape and orientation of the polygon that forms the

perimeter of the folded unit, with respect to the orientation of the Standard Basis. Creases whose angles are approximately factors of 90 when measured in degrees, including non-integer values, are all good places to start, since they evenly divide the corners and sides of the starting square of paper. Amongst these, 90, 45, 22.5, 60, 30 and 15-degree angles are most commonly used in my book, and in general in kusudama design, because they are the easiest referenced, and these are descriptively reducible to multiples of 22.5 and 15 degrees. Multiples of 22.5-degree angles divide evenly into a square and are easily obtained by halving the appropriate existing angles *when the starting* Standard Basis is rotationally oriented to one of the same multiples. For example, when the Baseline is rotated 22.5 degrees in either direction, the appropriate references to join any corner of the square to one of the reference creases needed to obtain a 22.5-degree rotated Baseline (see the first steps of the Augusta kusudama for an example of these), while pivoting around the rotated Baseline, will be multiples of the same angles. Finding these same 22.5-degree references would be more difficult if the starting Baseline rotation was 30 degrees, Likewise, multiples of 15-degree angles are obtained by a combination of a bisection and trisection of the appropriate corners, of either the paper, or of angles between the edge of the paper and a fold, or between two folds whose orientation with respect to each other at the point of their intersection is orthogonal.

Q: Once the initial Standard Basis orientation is established, how might the unit design process proceed?

A: Once an initial line of symmetry is established, creases which intersect these lines of symmetry can begin to constrain the magnitude of either the baseline or ridgeline, since 2 points of the Polygon Basis will lie on these intersections. Each pair of new creases will exponentially increase the potential for folding sequence variation, so precisely predicting how to proceed becomes in practice impossible. However, using the Huzita-Justin axioms for the range of potential referenced folds, the initial reference creases which contain the vertices of the Polygon Base which lie on either the Baseline or Ridgeline provide a range of options. First, the designer might conceive of the Standard Basis in terms of a vector field, and consider the range of perpendicular vectors (as in, the normal vectors) whose bases are translated from the origin to some point on an existing crease line, and make folds along

one such pair of lines with convenient reference points lying on other crease intersections, other crease edges, or some point(s) on the perimeter of the square of paper. Additional reference creases running across the paper may also function to partially truncate the existing Identity Polygon Base's perimeter. Second, noting the resulting right angles formed by creases made along vectors perpendicular to the initial reference creases, the designer might consider the vectors whose interior angles, when measured in degrees, with respect to their intersection with the previous reference creases, are factors of the right angles formed by the perpendicular vector. In other words, the right angles which are formed can receive angle bisections, trisections, tetrasections, etc. which extend to the edge of the paper in either direction, or which stop upon intersecting at least two other creases. In the case of Type 2 irregular units (where the folds respective of the Standard Basis are not "regular" as defined above), angular subdivisions may be more arbitrary, in that any set of folds which has the potential of maintaining point-reflection isometry through various additional folds (consider the way in which Raetia kusudama from *Mind-Blowing Kusudama Origami* does this) may be sufficient to subdivide the newly generated right angles. Third, at least some of the paper will in most units be specifically allocated for locking mechanisms. Consider the circle packing techniques and molecular triangularization utilized by other design processes for point generation to isolate tab and pocket regions. In all of this, many of the folds which develop the various components of the unit might in practice be quite simple and require little genuine preplanning.

Color change embellishments will result from an alteration of layers, either through the utilization of a specific base, or through alterations of mountain and valley folds around the perimeter of the starting square. Point embellishment can be formed through either redundancies in the locking mechanisms, or through the allocation of regions of the paper for molecule development (as per traditional circle packing design processes). Indeed, many of the design algorithms of representative origami can be reapplied to kusudama unit design, provided one accommodates the specific design constraints of unit design.

For additional complexity and variation, one might start with a classic base and then proceed with all the techniques described above. For example, one might simply Blintz the starting square and then proceed, treating the resulting smaller square as they would have treated the original square. The Shakespeare kusudama from *Mind-Blowing Kusudama Origami* utilizes such a technique. However, layers can quickly accumulate in such cases, and the extra layers of paper must be allocated prudently for best results.

Q: What constraints do the initial reference creases impose, in terms of the unit's design?

A: The reference creases will either directly or indirectly define the location of the points which define the size, shape and orientation of the polygon that forms the perimeter of the folded unit, with respect to the orientation of the Standard Basis. The orientation can only change reflectionally, which is to say, trivially, with respect to the Standard Basis (in which case the creases flip in their respective orthogonal quadrants) due to the constraint that ≥2 points of the polygon base must lie on the Baseline. In other words, the Polygon Base is not rotationally independent from the standard Basis can be reoriented upon the completion of the unit. Orientation aside, the starting reference creases will significantly influence both the shape and size of the Identity Polygon Base.

Q: What distinguishes Adaptability from Alternate Baseline Assemblies?

A: Technically, there are occasions when there exists no difference between the two. For example, the assembly of a unit along its Ridgeline is identical to assembling a unit where the Baseline is orthogonal to its original orientation, which is a 90-degree rotated Baseline. However, Adaptability highlights only 90-degree Baseline rotations and the two potential orientations of the unit: where the Baseline is oriented as a valley fold from the position of the observer, and where the Baseline is oriented as a mountain fold from the perspective of the observer. These are delineated as "Adaptability" because they are conceptually (though certainly not visually) trivial transformations, usually obtained through minimal effort, either through reversing the orientation of the central crease, the tabs, and the locks, or through folding the Ridgeline, which can be obtained by joining opposite vertices of the Baseline.

Alternate Baseline Assemblies utilize a central line, about which the creases of the unit have 2-fold rotational symmetry, as a new Baseline, which is rotated

with respect to the original orientation of the Baseline in the completed unit by some value between 0 and 180 degrees. Many of the details of kusudama unit design follow from the Adaptability of the units. For example, the magnitude of the vectors extending from the vertices of the Polygon Base which lie on the Baseline will define the coefficient difference between the maximum and minimum diameters of the completed assembly of any regular polyhedral structure, if and only if the Adaptability of the units facilitates mountain and valley orientations.

Q: Do the Baseline and Ridgeline as visualized on the original starting square extend to the edge of the paper in the completed unit? In other words, do these lines maintain the same magnitude through the folding process?

A: In some units, they will as physically folded creases, but in many others they will not. The units where lines constituting the Standard Basis extend uninterrupted to the edge of the paper on the completed unit are usually fairly simple and paper area-efficient units. "Uninterrupted" here means that no folded crease which constitutes part of the final crease pattern of the standard unit passes through or directly intersects either a) the Ridgeline or b) the Baseline, in such a way that more than a single layer of paper exists where these lines intersect the unit's perimeter. From my experience, the initial symmetry fixing the rotation of the Standard Basis with respect to the paper is identical to the final orientation in the case of such units, and no folded lines from the completed design tend to intersect the ridgeline, which entails that \geq 2 vertices of the Identity Polygon Base lie on the perimeter of the square. By contrast, in many unit designs, neither the Baseline nor Ridgeline will extend to the edge of the paper in the completed unit. This is also usually the case when the rotational symmetry of the Standard Basis with respect to the completed unit is altered in order to construct alternative assemblies. However, in the case of variations, the inverse may also be true: the creases forming the Baseline and Ridgeline may not extend uninterrupted by creases to the edge of the starting square, but altering the rotational orientation of the Standard Basis for variations may result in them doing so. For an example of this in *Mind-Blowing Kusudama Origami*, consider the Susiana kusudama. In standard units, no vertices of the hexagonal Polygon Base lie on the perimeter of the paper (this is quite easily illustrated by a glance at the crease pattern).

However, in the Susiana variant, the Polygon Base is altered when the Standard Basis is rotated, and this results in a more paper area-efficient unit whose ridgeline extends uninterrupted to the perimeter of the paper. Some units will not even fold the Standard Basis creases, although they still have them as a line of symmetry. Consider Ekaterina Lukasheva's Paradigma Kusudama for an example of this.

Q: What is meant by the "handedness" of a kusudama unit?

A: Kusudama units which possess an axis of rotational symmetry with two or more sides may also have a "handedness." The unit may be "right-handed" or "left-handed," and each of these is mathematically referred to as the other's enantiomorph. The handedness of the unit corresponds to reflections across a central axis. Practically, changing from one handedness to another would usually involve flipping all of the creases to the opposite side of the paper across a central axis. When this action is performed, the tabs which were allocated from paper in the upper left and lower right quadrant would be allocated from paper in the upper right and lower left quadrant, and the pockets which were allocated from paper in the upper left and lower right quadrant would be allocated from paper in the upper left and lower right quadrant. The visual effect of such transformations is almost universally negligible, but they illustrate the symmetry of the units. Also, with some folding sequences, making creases in a specific handedness may be more comfortable for the folder.

Q: How many different types of pockets/tabs are possible?

A: Fundamentally, in pure origami, the friction between two or more pieces of paper holds together kusudamas without the use of additional materials or adhesives. The range of the potential types of kusudama assemblies has never been exhaustively enumerated, but from my experience folding, I have noted several types. Note that these are neither mutually exclusive nor collectively exhaustive.

a) Dihedral overlaps: Dihedral overlap locks are often generated through inside reverse folds. Inside reverse fold locks are constituted by a pocket which is formed by an inside reverse fold, and some corresponding tab which is inserted between the upper and lower layers of the inside reverse fold. This tab may or may not fully occupy all of the available space within the pocket region, but may not be larger than the available space within the pocket, unless the entirety of the tab is purposefully not inserted into the pocket. While inside reverse folds may be used to generate many of the pocket types below, in this context, they refer specifically to locks which rely on the tabs of a unit sufficiently filling a pocket which extends beyond a Ridgeline, Baseline, or Polygon Base perimeter crease of an adjacent unit, and utilizes this extension to generate sufficient friction to hold adjacent units together by utilizing the dihedral angle between adjacent planes of different units in the completed assembly.

b) Layered slits: These types of pockets are created through a gap existing between multiple layers of paper. These layers may consist of multiple regions of the paper converging upon a single region associated with a pocket in a completed unit, or may be the product of the back-and-forth alternation of a single layer within the pocket region. In the case of such units, the tab is inserted between two layers of the slit in a manner similar to that of the Inside reverse folds described above. Note that this describes a unique type of lock, rather than a unique locking mechanism. Several of the other locking mechanisms may be used to hold paper within a layered slit.

c) Wrap arounds: These locks often invert the opening of the pocket, reflecting it across a line of symmetry identical to the edge of the polygon base corresponding to the pocket side of the assembly. Thus, the edge of the pocket in such cases is closer to the center of the unit, and the deepest part of the pocket is closer to the edge of the unit. They traditionally mountain fold the tab around and wrap it into a pocket layer underneath, often relying on the tension of the paper trapped between the outer edge of the opening of the pocket and the edge which separates the tab from the unit's Polygon Base. As an example from *Mind-Blowing Kusudama Origami*, consider Valdoria, which utilizes several locking mechanisms, including a wrap around.

d) Fold throughs: These locks align one or more folds of adjacent units with each other in such a way that one or more folds can be performed through two or more units simultaneously, thus locking them together. When multiple folds are performed to hold adjacent units together, they may be in alternating or identical orientations (as in, mountain or valley folds), and may be performed in a specific or non-specific order. For an example of a fold through lock with multiple folds performed in a specific order, consider the Ridgeline variant of the Aquitania kusudama from *Mind-Blowing Kusudama Origami*.

e) Friction locks: While all kusudama locks of pure modular origami rely on the friction of the paper, friction locks specifically refer to those in which strictly the tension between the layers of the paper holds the tab in the pocket. In other language, the surface of the tab and pocket regions of adjacent units remain planar, (as in, no folds pass through either the tab or the pocket to alter their dihedral angles from 180 degrees, and no curved, non-planar surfaces exist in the tabs or pockets). As an example of this type of lock, consider the Aquileia kusudama from *Mind-Blowing Kusudama Origami*. Also, the classic Sonobe unit would be an example of such a lock.

f) Capstone assemblies: These less-common types of locks rely upon the tension generated by the relationship between the placement of multiple pieces of paper in order to hold the whole structure together in situations where the lock between any two units alone would be insufficient to hold them together. Examples of models which utilize this type of lock in addition to other locking mechanisms from *Mind-Blowing Kusudama Origami* include Capstone and the Zeta Ridgeline assembly. The classic example of a model which relies solely on this locking mechanism is Kenneth Kawamura's Butterfly Ball.

g) Paper displacement/dispersion locks: These locks rely on separating one or more layers of paper of adjacent units, and positioning these sections of the units in such a way with respect to each other that the tension between them sufficiently holds them together. Examples of locking mechanisms which utilize such techniques in this category might include Toshikazu Kawasaki's twist locks and Ekaterina Lukasheva's Paradigma kusudama. Note that these are often very similar to Capstone assemblies.

Note that any combination of the above locks is conceivable in multi-step lock assemblies.

Q: How does the Polygon Base relate to the paper area efficiency of the unit?

A: They are directly proportional. The visual reference to the completed unit often used as a silhouette superimposed upon the starting square of paper is in most case identical with the unit's Augmented Polygon Base.

Q: What determines the shape and the size of the Polygon Base?

A: Although four is the minimum number of vertex points to define a nondegenerate Polygon Base edge unit, the number of points on the Polygon Base without the aforementioned regions external to the central polygon rarely exceeds six to eight, except in cases where concave exterior facets are added to the Augmented Polygon Base. Augmented Polygon bases with concave facets frequently have >8 sides. Caria would be an example of this. Why do Identity Polygon Bases rarely exceed 8 vertices? A simplified answer is that additional polygonal sides function as a design challenge: as more sides are added, the function and placement of those sides within the polyhedral structure must be established. Sometimes additional sides will have an additional set of corresponding tabs and pockets, which assemble separately from the rest of the unit, but this adds complexity to the design. Likewise, the addition of extra polygonal sides in an Identity Base requires additional folds to establish the reference constraints of the polygonal vertices. Perhaps most importantly is that the starting size of the paper is assumed as a fixed parameter, which means that as more sides are added to the Identity Polygon Base (which, as a reminder, must be convex due to the rules of flat-foldability), the base will take up a greater proportion of the starting square's available area, or alternatively, the sides of the base will shrink in length, resulting in the Identity Base being scaled down with respect to the paper. Since pockets and tabs usually correspond to one or more sides of a Polygon Base, this means the pockets and tabs will be smaller, and thus more difficult to design as the number of sides increase (because the regions of paper allocated for the tabs and pockets will usually have to be increasingly isolated regions from each other, separated from other regions by more surrounding paper) and to assemble (because the locking assembly regions of the paper will represent a smaller proportion of the paper). The Augmented Polygon Base can include a variety of concave angles, and can also be expanded to include areas of the paper that are separated from the center polygon by one or more creases (which are in the reverse orientation of the perimeter of the polygon base) in a completed model. This means that the Augmented Base includes fewer

constraints, but the design process for Augmented Bases is less straightforward. Throughout this Q&A, I reference "Polygon Base" generally, or "Identity Polygon Base/Augmented Polygon Base" as deemed appropriate.

Q: Why does any Polygon Base constituting an edge unit which is representative of the edge of a polyhedral structure need to have an even number of sides?

A: The edge unit which represents the entirety of an edge must have 2-fold rotational symmetry in order to model symmetric polyhedra. No polygons with an odd number of sides have 2-fold rotational symmetry. However, any two odd numbers added together yield an even number, so it is entirely conceivable that units whose Polygon Bases have an odd number of sides can be assembled back-to-back to constitute a single "macro unit," which then has 2-fold symmetry. Also, some units have an even number of sides, but do not have 2-fold rotational symmetry. None of these types of unit are diagrammed in *Mind-Blowing Kusudama Origami*, but many such designs exist elsewhere.

Q: Do all kusudamas model regular polyhedra?

A: No, but most do. Non-regular compositions usually involve more than one type of unit.

Q: Does the range of modelable polyhedra (that is, the 3D structures which are capable of being assembled out of kusudama units) include constructions with more than one edge length?

A: The edge of the polyhedron corresponds to the Baseline of the unit. If the polyhedron includes more than one edge length, the Baselines will have to scale proportionally. Since kusudamas are traditionally composed of a single type of unit (with the exception that many kusudamas include separately folded additional inserts and embellishments which are added to the composition), they will have a Baseline of a single length, and thus will not be capable of modeling polyhedral structures with more than one edge length. Polyhedra which are capable of modification to have equal length edges are in theory capable of being modeled using kusudama units, but the limits on the assembly of any given unit design will depend upon other factors of the unit design as highlighted in the Adaptability section of each unit.

Q: Does the Baseline of the unit need to lie directly on the edge of the underlying polyhedral assembly structure?

A: No, not in all cases. Units with an expanded Augmented Polygon Base often include sections of the paper which displace the Baselines away from each other in a manner that they will no longer align with the underlying rotational orientation of the edges with respect to each other in the polyhedron. In many cases, windows which form around the vertices of the Baseline do not result in the Baselines being rotated off of their underlying polyhedron. In these cases, the Baseline is a proportional scalar multiple of the edge of the underlying polyhedron (as in, the Baseline will be some fraction of the length of the edge in the underlying polyhedron edge, and will move in an identical direction to that edge). However, in the case of nearly all closed kusudamas which possess no windowed regions, and which model regular polyhedra, the Baselines will line up exactly with an underlying polyhedral symmetry.

Q: What is the range of potential rotations?

A: The first type of potential rotation occurs when both the Polygon Base and the Baseline/Ridgeline (where the latter is referred to as the Standard Basis) are rotated together so that their orientation with respect to each other is fixed. The second type occurs when the Polygon Base is rotated with respect to the paper, whilst the Standard Basis is fixed. The third type occurs when the Standard Basis rotates with respect to the paper, while the Polygon Basis remains fixed. Finally, the Standard Basis may be rotated in a completed unit to align with regions of the paper beyond the Polygon Base. This may be equivalent to the rotation of the Polygon Base and the Standard Basis independent of each other with respect to the paper. Note also that all rotations may be performed in either a clockwise or counterclockwise direction.

Q: What is meant by "irregular unit," and "irregularization?"

A: For the sake of conceptual simplicity, *Mind-Blowing Kusudama Origami* classifies the angles of the creases within a kusudama unit that are multiples of 15 and 22.5 degrees as "regular." This is because these angles are easily obtained factors of 90 and 180 degrees, which, as a reminder, represent the interior angles of the corners of the squares and the angles formed around any point placed along the edges of the square. Given that there exist two potential

degrees of rotational freedom, as well as the potential for a vast variety of 2fold symmetric irregular polygons which may constitute the Polygon Bases, several forms of "irregular units" are possible. First, one should note that irregular units are not intrinsically correlated to complexity, as many "irregular units" are quite simple, (the Monarchy Star from *Mind-Blowing* Kusudama Origami would be an example of such a unit), and many "regular" units can be extremely complex (consider units folded with a box pleated base, or consider some of the more complicated 15-degree derivative crease pattern units from *Mind-Blowing Kusudama Origami*). Regarding the differing types of irregular units, the first type possesses a regular Identity Polygon Base, here defined simply as a region of the paper which contains a regular polygon. This base is then rotated in its orientation with respect to the paper, resulting in a set of creases which are irregular with respect to the paper and initial reference creases (which rely on the initial symmetry of the paper). A second type of irregular unit is one in which the creases forming the polygon basis are irregular (as defined above) respective of the Baseline and Ridgeline, regardless of the orientation of these lines with respect to the paper. This results in an irregular Polygon Base. A third type of units combine both of these irregularities together, which results in an irregular Polygon Base oriented at an irregular rotation with respect to the paper. Note that some crease patterns combine both an irregular angled Polygon Base with an irregular rotation to generate a set of reference creases which, as a result of the two distinct "irregularizations" cancelling each other out, are regular with respect to the paper. Irregularization in my book often refers to processes within the unit design sequence which harness the potential of such units.

Q: What is the purpose of designing irregular crease patterns?

A: There is no express purpose for using irregular crease patterns. Utilizing irregular creases simply removes the constraints of regular angles and opens up a wider array of options.

Q: How does the size of the unit correlate to the size of the completed kusudama?

A: The Baseline magnitude in conjunction with the polyhedron which the units are modeling establishes the approximate diameter of the completed kusudama. It is quite easily demonstrated that changing the orientation of the units' Baselines from valley to mountain orientation will not alter the diameter of the sphere inscribed by the Baselines of the units, and thus a mountain orientation will generally have a smaller diameter.

Q: How might this design process be succinctly summed up?

A: Consider in advance the type of base desired, and how standard circle packing might establish the vertices of the Polygon Bases within the constraints of a square. Establish a set of reference creases. Use these to establish a line of symmetry. Use this to add framing creases, add angular subdivisions between creases, bring together multiple alternately mountain and valley-oriented folds to converge along the perimeter of the Polygon Base. Consider how existing layering patterns could be either used or modified in order to create tab and pocket regions which maintain the symmetry stipulations described above, and how the tab regions relate to the pocket regions. Finally, consider how various layers can contribute to the exterior visual effects of the unit, and how these might contribute to or interfere with the assembly process of the units.